MULTIMEDIA UNIVERSITY

STUDENT IDENTIFICATION NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

BMS2024 -ADVANCED MANAGERIAL STATISTICS

(All Sections / Groups)

19 OCTOBER 2019
2.30 pm – 4.30 pm
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 15 pages **excluding** the cover page.
2. This question paper consists of **FOUR** structured questions. Attempt **ALL** questions.
3. Students are allowed to use non-programmable scientific calculators with no restrictions.
4. A formulae list and statistical tables are attached at the end of the question paper.
5. Use **pen** to write the answers in the answer booklet provided.

QUESTION 1 (25 Marks)

Spam e-mail has become a serious and costly nuisance. An office manager believes that the average amount of time spent by office workers reading and deleting spam exceeds 25 minutes per day. He takes a random sample of 18 workers and measures the amount time each spends reading and deleting spam. The sample mean is 30 minutes. The population standard deviation is 12 minutes. Can the manager infer that he is correct?

a) Establish the appropriate null and alternative hypotheses. (2 marks)

b) Compute the test statistic and the p-value of the test. (6 marks)

c) Based on the p-value obtained in (b), what is your statistical decision? Use $\alpha = 0.01$. (3 marks)

d) State the possible Type I and Type II errors that might occur. (4 marks)

e) At 0.05 level of significance, compute the probability of a Type II error. Given that the actual average amount of time spent reading and deleting spam is 28 minutes. (7 marks)

f) Compute the power of the test. (2 marks)

g) If the significance level is decreased, what is the effect on the power of the test. (1 mark)

Continued...

QUESTION 2 (25 Marks)

a) State **three assumptions** and **one limitation** for Independence *t*-test.

(5 marks)

b) A perfume manufacturer is trying to choose between two magazine advertising layouts. An expensive layout would include a small package of the perfume. A cheaper layout would include a 'scratch and sniff' sample of the product. The manufacturer would use the more expensive layout only if there is evidence that would lead to a higher approval rate.

The manufacturer presents the more expensive layout to four groups and determines the approval rating for each group. He presents the 'scratch and sniff' layout to five groups and again, determines the approval rating of the perfume for each group. Apply an appropriate statistical test for the listed data below at a level of significance of 0.05. Assume the listed data is non normally distributed:

Package	52	68	43	48

Scratch	37	40	53	39	47

(20 marks)

Continued...

QUESTION 3 (25 Marks)

Bob Stark is conducting research on monthly expenses for medical care, including over the counter medicine. His dependent variable is monthly expenses ($) for medical care while the independent variables are number of family members, life insurance ($) and health insurance ($). The summary output of the analysis is shown below:

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	132875933	43977657	48.48745	$1.21*10^{-7}$
Residual	14	126978660	906990.4		
Total	17	322909753			
	Coefficients	*Std Error*	*t Stat*	*P-value*	
Intercept	144.91	1025.911	0.141246		
Family	11.63	1.247247	9.330762	$2.19*10^{-4}$	
Life	13.70	8.786907	-1.55916	0.141272	
Health	-9.11	1.166068	7.810781	$1.81*10^{-6}$	

a) State the multiple linear regression equation for the above data. (4 marks)

b) Interpret the slope coefficient for the number of family members and health insurance relating to the monthly expenses for medical care. (4 marks)

c) Compute the coefficient of multiple determination. Interpret the value. (4 marks)

d) At the 5 percent level of significance, test the overall validity of the model. (4 marks)

e) At the 1 percent level of significance, test if each independent variable is significantly related towards the monthly expenses for medical care. (6 marks)

f) Determine Bob Stark's monthly expenses for his medical care if the family members are 7 persons, insured $1377 for his life insurance and insured $953 of his health insurance. (3 marks)

Continued...

QUESTION 4 (25 Marks)

During a study, individuals were asked to rate a product on a scale of 1-5. From the following summary output, help the researcher determine whether any significant differences exist in opinions among individuals from different regions: South, North and East. Assume that the dataset is normally distributed.

Summary Output

Groups	Count	Sum	Mean	Variance
South	7	23	3.29	2.2381
North	10	29	2.90	2.3222
East	8	23	2.88	2.6964

ANOVA

Source of Variation	SS	df	MS	F
Among Groups	0.80	2	0.40	0.1653
Within Groups	53.20	22	2.42	
Total	54	24		

a) At the 5 percent level of significance, is there evidence of a difference in the mean exist of individually opinions from the three regions regarding the product. Conduct an appropriate statistical procedure. (10 marks)

b) Conduct the Tukey-Kramer post-hoc test to examine which region differ in mean rating the product. Use 10 percent significance level. (15 marks)

End of Paper

STATISTICAL FORMULAE

A. DESCRIPTIVE STATISTICS

Sample Mean = $\overline{X} = \dfrac{\sum_{i=1}^{n} X_i}{n}$ Sample Standard Deviation = s = $\sqrt{\dfrac{\sum_{i=1}^{n} X_i^2}{n-1} - \dfrac{\left(\sum_{i=1}^{n} X_i\right)^2}{n(n-1)}}$

where n = *number of observations*
X_i = *the* i^{th} *observation of the data*

B. HYPOTHESIS TESTING

Types of Error
Type I Error = α= P(Rejecting H_0 | H_0 is true) where, Confidence Interval = 1 - α
Type II Error = β= P(Not Rejecting H_0 | H_0 is false)

One Sample Mean Test

σ Known	σ Unknown
$z = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$	$t = \dfrac{\overline{x} - \mu}{s / \sqrt{n}}$

Two Sample Mean Test

Comparing Means for Two Independent Populations

[Standard Deviation (σ) Known]	[Standard Deviation (σ) Not Known]
$z = \dfrac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}}$	$t = \dfrac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(1/n_1 + 1/n_2\right)}}$ where $S_p^2 = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 + n_2 - 2)}$

Two Sample Mean Test

Comparing Means for Two Paired Populations

$t = \dfrac{(\overline{D} - \mu_D)}{S_D / \sqrt{n}}$ where $\overline{D} = \dfrac{\sum_{i=1}^{n} D_i}{n}$ and $S_D = \sqrt{\dfrac{\sum_{i=1}^{n} D_i^2}{n-1} - \dfrac{\left(\sum_{i=1}^{n} D_i\right)^2}{n(n-1)}}$

Non-Parametric Analysis

Wilcoxon Rank Sum Test	**Wilcoxon Signed Rank Sum Test**
$Z = \dfrac{(T_1 - \mu_{T_1})}{\sigma_{T_1}}$ where	$Z = \dfrac{(T_+ - \mu_{T_+})}{\sigma_{T_+}}$ where
$\mu_{T1} = \dfrac{n_1(n+1)}{2}$ and	$\mu_{T+} = \dfrac{n(n+1)}{4}$ and
$\sigma_{T_1} = \sqrt{\dfrac{n_1 n_2 (n+1)}{12}}$ where $n = n_1 + n_2$	$\sigma_{T_+} = \sqrt{\dfrac{n(n+1)(2n+1)}{24}}$

Kruskal-Wallis Rank Test

$$H = \left[\frac{12}{n(n+1)}\sum_{j=1}^{c}\frac{T_j^2}{n_j}\right] - 3(n+1)$$ where the critical value is χ^2 with $df = c - 1$

Check ranking sum: $\sum T_j = n(n+1)/2$

Chi-Square Test

$$\chi^2 = \sum^{n}\frac{(O-E)^2}{E}$$

where O = Frequency of Observed Values *and* *E = Frequency of Expected Values*

with $df = c - 1$ *where c = number of categories*
or
with $df = (r-1)(c-1)$ *where r = number of rows and c = number of columns*

C. ANALYSIS OF VARIANCE (ANOVA)

One-Way ANOVA

Source	**Degrees of Freedom**	**Sum of Squares**	**Mean Squares**	**F-statistic**
Among Groups	*c - 1*	SSA	MSA = SSA/*c-1*	MSA/MSW
Within Groups	*n - c*	SSW	MSW = SSW/*n-c*	
Total	*n - 1*	SST		

$$SST = \sum_{j=1}^{c}\sum_{i=1}^{n_j}\left(X_{ij} - \overline{\overline{X}}\right)^2$$ *or alternative formula:* $$SST = \left(\sum_{j=1}^{c}\sum_{i=1}^{n_j}X_{ij}^2\right) - \frac{\left(\sum_{j=1}^{c}\sum_{i=1}^{n_j}X_{ij}\right)^2}{n}$$

$$SSA = \sum_{j=1}^{c} n_j\left(\overline{X}_j - \overline{\overline{X}}\right)^2$$ *and SSW = SST - SSA*

where n = number of observations, c = number of groups and $\overline{\overline{X}}$ *= overall mean*

Tukey-Kramer Procedure

Critical Range = $Q_U\sqrt{\frac{MSW}{2}\left[\frac{1}{n_i}+\frac{1}{n_j}\right]}$

where Q_u = the upper tail critical value from a Studentized Range Distribution having (c) degrees of freedom in the numerator and (n – c) degrees of freedom in the denominator at a given level of significance

Two-Way ANOVA

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F-statistic
A	$r-1$	SSA	MSA = SSA/$(r-1)$	MSA / MSE
B	$c-1$	SSB	MSB = SSB/$(c-1)$	MSB / MSE
AB	$(r-1)(c-1)$	SSAB	MSAB = SSAB/$(r-1)(c-1)$	MSAB / MSE
Error	$rc\,(n-1)$	SSE	MSE = SSE/$rc(n'-1)$	
Total	$n-1$	SST		

where,

Factor A levels are represented by the rows and Factor B levels are represented by the columns and
n = number of observations
c = number of columns
r = number of rows
n' = number of replicates

$$SST=\sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n'}\left(X_{ijk}-\overline{\overline{X}}\right)^2 \qquad SSA=cn'\sum_{i=1}^{r}\left(\overline{X}_i-\overline{\overline{X}}\right)^2$$

$$SSB=rn'\sum_{j=1}^{c}\left(\overline{X}j-\overline{\overline{X}}\right)^2$$ where $\overline{\overline{X}}$ = *overall mean*

$$SSE=(n'-1)\left[S_1^2+S_2^2+\dots\dots+S_k^2\right]$$ where S_i^2 = variance of each block

D. REGRESSION ANALYSIS

Multiple Linear Regression

Population Model: $Y=\beta_0+\beta_1X_1+\beta_2X_2+\dots\dots+\beta_kX_k+\varepsilon$

Sample Model: $\hat{y}=b_0+b_1x_1+b_2x_2+\dots\dots+b_kx_k+e$

Adjusted R-Square $=1-\left[\frac{(1-R^2)(n-1)}{(n-p-1)}\right]$ where p = number of independent/predictor variables

ANOVA Table for Regression			
Source	**Degrees of Freedom**	**Sum of Squares**	**Mean Squares**
Regression	p	SSR	MSR = SSR/p
Error/Residual	$n-p-1$	SSE	MSE = SSE/$(n-p-1)$
Total	$n-1$	SST	

Test Statistic for Significance of the Overall Regression Model

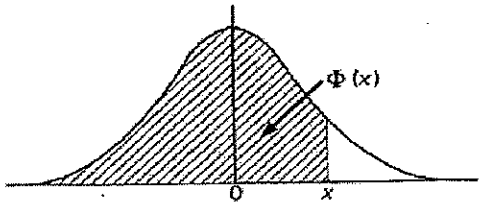
F = MSR/MSE

Test Statistic for Significance of Each Predictor Variable

$t_i = \dfrac{b_i}{S_{b_i}}$ and the critical value $= \pm t_{\alpha/2,(n-p-1)}$

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
			0·9994		0·99990		0·99995
3·075		3·263		3·731		3·916	
	0·9990		0·9995		0·99991		0·99996
3·105		3·320		3·759		3·976	
	0·9991		0·9996		0·99992		0·99997
3·138		3·389		3·791		4·055	
	0·9992		0·9997		0·99993		0·99998
3·174		3·480		3·826		4·173	
	0·9993		0·9998		0·99994		0·99999
3·215		3·615		3·867		4·417	
	0·9994		0·9999		0·99995		1·00000

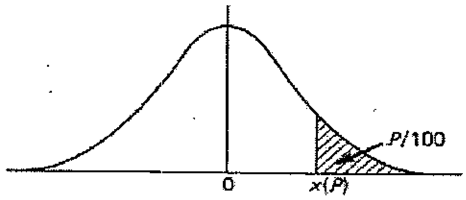
When $x > 3·3$ the formula $1-\Phi(x) \doteqdot \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

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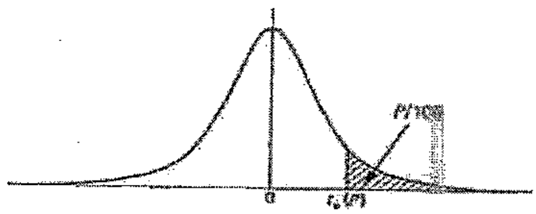
TABLE 10. PERCENTAGE POINTS OF THE t-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}}\frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)}\int_{t_\nu(P)}^{\infty}\frac{dt}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2-distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t-distribution with ν degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

P	40	30	25	20	15	10	5	2·5	1	0·5	0·1	0·05
ν = 1	0·3249	0·7265	1·0000	1·3764	1·963	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	0·2887	0·6172	0·8165	1·0607	1·386	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	0·2767	0·5844	0·7649	0·9785	1·250	1·638	2·353	3·182	4·541	5·841	10·21	12·92
4	0·2707	0·5686	0·7407	0·9410	1·190	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	0·2672	0·5594	0·7267	0·9195	1·156	1·476	2·015	2·571	3·365	4·032	5·893	6·869
6	0·2648	0·5534	0·7176	0·9057	1·134	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	0·2632	0·5491	0·7111	0·8960	1·119	1·415	1·895	2·365	2·998	3·499	4·785	5·408
8	0·2619	0·5459	0·7064	0·8889	1·108	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	0·2610	0·5435	0·7027	0·8834	1·100	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	0·2602	0·5415	0·6998	0·8791	1·093	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	0·2596	0·5399	0·6974	0·8755	1·088	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	0·2590	0·5386	0·6955	0·8726	1·083	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	0·2586	0·5375	0·6938	0·8702	1·079	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	0·2582	0·5366	0·6924	0·8681	1·076	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	0·2579	0·5357	0·6912	0·8662	1·074	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	0·2576	0·5350	0·6901	0·8647	1·071	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	0·2573	0·5344	0·6892	0·8633	1·069	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	0·2571	0·5338	0·6884	0·8620	1·067	1·330	1·734	2·101	2·552	2·878	3·610	3·922
19	0·2569	0·5333	0·6876	0·8610	1·066	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	0·2567	0·5329	0·6870	0·8600	1·064	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	0·2566	0·5325	0·6864	0·8591	1·063	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	0·2564	0·5321	0·6858	0·8583	1·061	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	0·2563	0·5317	0·6853	0·8575	1·060	1·319	1·714	2·069	2·500	2·807	3·485	3·768
24	0·2562	0·5314	0·6848	0·8569	1·059	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	0·2561	0·5312	0·6844	0·8562	1·058	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	0·2560	0·5309	0·6840	0·8557	1·058	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	0·2559	0·5306	0·6837	0·8551	1·057	1·314	1·703	2·052	2·473	2·771	3·421	3·690
28	0·2558	0·5304	0·6834	0·8546	1·056	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	0·2557	0·5302	0·6830	0·8542	1·055	1·311	1·699	2·045	2·462	2·756	3·396	3·659
30	0·2556	0·5300	0·6828	0·8538	1·055	1·310	1·697	2·042	2·457	2·750	3·385	3·646
32	0·2555	0·5297	0·6822	0·8530	1·054	1·309	1·694	2·037	2·449	2·738	3·365	3·622
34	0·2553	0·5294	0·6818	0·8523	1·052	1·307	1·691	2·032	2·441	2·728	3·348	3·601
36	0·2552	0·5291	0·6814	0·8517	1·052	1·306	1·688	2·028	2·434	2·719	3·333	3·582
38	0·2551	0·5288	0·6810	0·8512	1·051	1·304	1·686	2·024	2·429	2·712	3·319	3·566
40	0·2550	0·5286	0·6807	0·8507	1·050	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	0·2547	0·5278	0·6794	0·8489	1·047	1·299	1·676	2·009	2·403	2·678	3·261	3·496
60	0·2545	0·5272	0·6786	0·8477	1·045	1·296	1·671	2·000	2·390	2·660	3·232	3·460
120	0·2539	0·5258	0·6765	0·8446	1·041	1·289	1·658	1·980	2·358	2·617	3·160	3·373
∞	0·2533	0·5244	0·6745	0·8416	1·036	1·282	1·645	1·960	2·326	2·576	3·090	3·291

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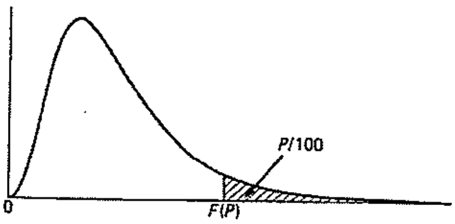
TABLE 12(*a*). 10 PER CENT POINTS OF THE *F*-DISTRIBUTION

The function tabulated is $F(P) = F(P|\nu_1, \nu_2)$ defined by the equation

$$\frac{P}{100} = \frac{\Gamma(\frac{1}{2}\nu_1 + \frac{1}{2}\nu_2)}{\Gamma(\frac{1}{2}\nu_1)\,\Gamma(\frac{1}{2}\nu_2)}\,\nu_1^{\frac{1}{2}\nu_1}\,\nu_2^{\frac{1}{2}\nu_2}\int_{F(P)}^{\infty}\frac{F^{\frac{1}{2}\nu_1 - 1}}{(\nu_2 + \nu_1 F)^{\frac{1}{2}(\nu_1+\nu_2)}}\,dF,$$

for $P = 10$, 5, 2·5, 1, 0·5 and 0·1. The lower percentage points, that is the values $F'(P) = F'(P|\nu_1, \nu_2)$ such that the probability that $F \leqslant F'(P)$ is equal to $P/100$, may be found by the formula

$$F'(P|\nu_1, \nu_2) = 1/F(P|\nu_2, \nu_1).$$



(This shape applies only when $\nu_1 \geqslant 3$. When $\nu_1 < 3$ the mode is at the origin.)

ν_2 \ $\nu_1 =$	1	2	3	4	5	6	7	8	10	12	24	∞
$\nu_2 =$ 1	39·86	49·50	53·59	55·83	57·24	58·20	58·91	59·44	60·19	60·71	62·00	63·33
2	8·526	9·000	9·162	9·243	9·293	9·326	9·349	9·367	9·392	9·408	9·450	9·491
3	5·538	5·462	5·391	5·343	5·309	5·285	5·266	5·252	5·230	5·216	5·176	5·134
4	4·545	4·325	4·191	4·107	4·051	4·010	3·979	3·955	3·920	3·896	3·831	3·761
5	4·060	3·780	3·619	3·520	3·453	3·405	3·368	3·339	3·297	3·268	3·191	3·105
6	3·776	3·463	3·289	3·181	3·108	3·055	3·014	2·983	2·937	2·905	2·818	2·722
7	3·589	3·257	3·074	2·961	2·883	2·827	2·785	2·752	2·703	2·668	2·575	2·471
8	3·458	3·113	2·924	2·806	2·726	2·668	2·624	2·589	2·538	2·502	2·404	2·293
9	3·360	3·006	2·813	2·693	2·611	2·551	2·505	2·469	2·416	2·379	2·277	2·159
10	3·285	2·924	2·728	2·605	2·522	2·461	2·414	2·377	2·323	2·284	2·178	2·055
11	3·225	2·860	2·660	2·536	2·451	2·389	2·342	2·304	2·248	2·209	2·100	1·972
12	3·177	2·807	2·606	2·480	2·394	2·331	2·283	2·245	2·188	2·147	2·036	1·904
13	3·136	2·763	2·560	2·434	2·347	2·283	2·234	2·195	2·138	2·097	1·983	1·846
14	3·102	2·726	2·522	2·395	2·307	2·243	2·193	2·154	2·095	2·054	1·938	1·797
15	3·073	2·695	2·490	2·361	2·273	2·208	2·158	2·119	2·059	2·017	1·899	1·755
16	3·048	2·668	2·462	2·333	2·244	2·178	2·128	2·088	2·028	1·985	1·866	1·718
17	3·026	2·645	2·437	2·308	2·218	2·152	2·102	2·061	2·001	1·958	1·836	1·686
18	3·007	2·624	2·416	2·286	2·196	2·130	2·079	2·038	1·977	1·933	1·810	1·657
19	2·990	2·606	2·397	2·266	2·176	2·109	2·058	2·017	1·956	1·912	1·787	1·631
20	2·975	2·589	2·380	2·249	2·158	2·091	2·040	1·999	1·937	1·892	1·767	1·607
21	2·961	2·575	2·365	2·233	2·142	2·075	2·023	1·982	1·920	1·875	1·748	1·586
22	2·949	2·561	2·351	2·219	2·128	2·060	2·008	1·967	1·904	1·859	1·731	1·567
23	2·937	2·549	2·339	2·207	2·115	2·047	1·995	1·953	1·890	1·845	1·716	1·549
24	2·927	2·538	2·327	2·195	2·103	2·035	1·983	1·941	1·877	1·832	1·702	1·533
25	2·918	2·528	2·317	2·184	2·092	2·024	1·971	1·929	1·866	1·820	1·689	1·518
26	2·909	2·519	2·307	2·174	2·082	2·014	1·961	1·919	1·855	1·809	1·677	1·504
27	2·901	2·511	2·299	2·165	2·073	2·005	1·952	1·909	1·845	1·799	1·666	1·491
28	2·894	2·503	2·291	2·157	2·064	1·996	1·943	1·900	1·836	1·790	1·656	1·478
29	2·887	2·495	2·283	2·149	2·057	1·988	1·935	1·892	1·827	1·781	1·647	1·467
30	2·881	2·489	2·276	2·142	2·049	1·980	1·927	1·884	1·819	1·773	1·638	1·456
32	2·869	2·477	2·263	2·129	2·036	1·967	1·913	1·870	1·805	1·758	1·622	1·437
34	2·859	2·466	2·252	2·118	2·024	1·955	1·901	1·858	1·793	1·745	1·608	1·419
36	2·850	2·456	2·243	2·108	2·014	1·945	1·891	1·847	1·781	1·734	1·595	1·404
38	2·842	2·448	2·234	2·099	2·005	1·935	1·881	1·838	1·772	1·724	1·584	1·390
40	2·835	2·440	2·226	2·091	1·997	1·927	1·873	1·829	1·763	1·715	1·574	1·377
60	2·791	2·393	2·177	2·041	1·946	1·875	1·819	1·775	1·707	1·657	1·511	1·291
120	2·748	2·347	2·130	1·992	1·896	1·824	1·767	1·722	1·652	1·601	1·447	1·193
∞	2·706	2·303	2·084	1·945	1·847	1·774	1·717	1·670	1·599	1·546	1·383	1·000

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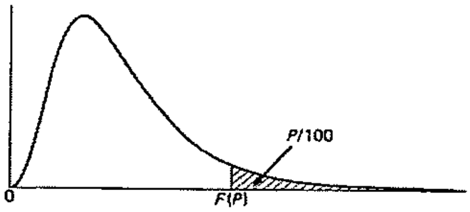
TABLE 12(*b*). 5 PER CENT POINTS OF THE *F*-DISTRIBUTION

If $F = \frac{X_1}{\nu_1} \Big/ \frac{X_2}{\nu_2}$, where X_1 and X_2 are independent random variables distributed as χ^2 with ν_1 and ν_2 degrees of freedom respectively, then the probabilities that $F \geqslant F(P)$ and that $F \leqslant F'(P)$ are both equal to $P/100$. Linear interpolation in ν_1 and ν_2 will generally be sufficiently accurate except when either $\nu_1 > 12$ or $\nu_2 > 40$, when harmonic interpolation should be used.



(This shape applies only when $\nu_1 \geqslant 3$. When $\nu_1 < 3$ the mode is at the origin.)

$\nu_1 =$	1	2	3	4	5	6	7	8	10	12	24	∞
$\nu_2 = 1$	161·4	199·5	215·7	224·6	230·2	234·0	236·8	238·9	241·9	243·9	249·1	254·3
2	18·51	19·00	19·16	19·25	19·30	19·33	19·35	19·37	19·40	19·41	19·45	19·50
3	10·13	9·552	9·277	9·117	9·013	8·941	8·887	8·845	8·786	8·745	8·639	8·526
4	7·709	6·944	6·591	6·388	6·256	6·163	6·094	6·041	5·964	5·912	5·774	5·628
5	6·608	5·786	5·409	5·192	5·050	4·950	4·876	4·818	4·735	4·678	4·527	4·365
6	5·987	5·143	4·757	4·534	4·387	4·284	4·207	4·147	4·060	4·000	3·841	3·669
7	5·591	4·737	4·347	4·120	3·972	3·866	3·787	3·726	3·637	3·575	3·410	3·230
8	5·318	4·459	4·066	3·838	3·687	3·581	3·500	3·438	3·347	3·284	3·115	2·928
9	5·117	4·256	3·863	3·633	3·482	3·374	3·293	3·230	3·137	3·073	2·900	2·707
10	4·965	4·103	3·708	3·478	3·326	3·217	3·135	3·072	2·978	2·913	2·737	2·538
11	4·844	3·982	3·587	3·357	3·204	3·095	3·012	2·948	2·854	2·788	2·609	2·404
12	4·747	3·885	3·490	3·259	3·106	2·996	2·913	2·849	2·753	2·687	2·505	2·296
13	4·667	3·806	3·411	3·179	3·025	2·915	2·832	2·767	2·671	2·604	2·420	2·206
14	4·600	3·739	3·344	3·112	2·958	2·848	2·764	2·699	2·602	2·534	2·349	2·131
15	4·543	3·682	3·287	3·056	2·901	2·790	2·707	2·641	2·544	2·475	2·288	2·066
16	4·494	3·634	3·239	3·007	2·852	2·741	2·657	2·591	2·494	2·425	2·235	2·010
17	4·451	3·592	3·197	2·965	2·810	2·699	2·614	2·548	2·450	2·381	2·190	1·960
18	4·414	3·555	3·160	2·928	2·773	2·661	2·577	2·510	2·412	2·342	2·150	1·917
19	4·381	3·522	3·127	2·895	2·740	2·628	2·544	2·477	2·378	2·308	2·114	1·878
20	4·351	3·493	3·098	2·866	2·711	2·599	2·514	2·447	2·348	2·278	2·082	1·843
21	4·325	3·467	3·072	2·840	2·685	2·573	2·488	2·420	2·321	2·250	2·054	1·812
22	4·301	3·443	3·049	2·817	2·661	2·549	2·464	2·397	2·297	2·226	2·028	1·783
23	4·279	3·422	3·028	2·796	2·640	2·528	2·442	2·375	2·275	2·204	2·005	1·757
24	4·260	3·403	3·009	2·776	2·621	2·508	2·423	2·355	2·255	2·183	1·984	1·733
25	4·242	3·385	2·991	2·759	2·603	2·490	2·405	2·337	2·236	2·165	1·964	1·711
26	4·225	3·369	2·975	2·743	2·587	2·474	2·388	2·321	2·220	2·148	1·946	1·691
27	4·210	3·354	2·960	2·728	2·572	2·459	2·373	2·305	2·204	2·132	1·930	1·672
28	4·196	3·340	2·947	2·714	2·558	2·445	2·359	2·291	2·190	2·118	1·915	1·654
29	4·183	3·328	2·934	2·701	2·545	2·432	2·346	2·278	2·177	2·104	1·901	1·638
30	4·171	3·316	2·922	2·690	2·534	2·421	2·334	2·266	2·165	2·092	1·887	1·622
32	4·149	3·295	2·901	2·668	2·512	2·399	2·313	2·244	2·142	2·070	1·864	1·594
34	4·130	3·276	2·883	2·650	2·494	2·380	2·294	2·225	2·123	2·050	1·843	1·569
36	4·113	3·259	2·866	2·634	2·477	2·364	2·277	2·209	2·106	2·033	1·824	1·547
38	4·098	3·245	2·852	2·619	2·463	2·349	2·262	2·194	2·091	2·017	1·808	1·527
40	4·085	3·232	2·839	2·606	2·449	2·336	2·249	2·180	2·077	2·003	1·793	1·509
60	4·001	3·150	2·758	2·525	2·368	2·254	2·167	2·097	1·993	1·917	1·700	1·389
120	3·920	3·072	2·680	2·447	2·290	2·175	2·087	2·016	1·910	1·834	1·608	1·254
∞	3·841	2·996	2·605	2·372	2·214	2·099	2·010	1·938	1·831	1·752	1·517	1·000

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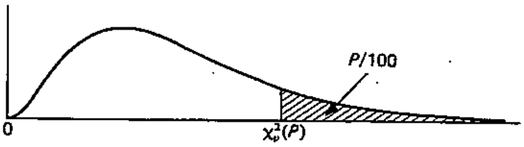
TABLE 8. PERCENTAGE POINTS OF THE χ^2-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2}\Gamma(\frac{\nu}{2})}\int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu-1} e^{-\frac{1}{2}x}\,dx.$$

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu - 1}$ and unit variance.



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

P	50	40	30	20	10	5	2.5	1	0.5	0.1	0.05
$\nu = 1$	0.4549	0.7083	1.074	1.642	2.706	3.841	5.024	6.635	7.879	10.83	12.12
2	1.386	1.833	2.408	3.219	4.605	5.991	7.378	9.210	10.60	13.82	15.20
3	2.366	2.946	3.665	4.642	6.251	7.815	9.348	11.34	12.84	16.27	17.73
4	3.357	4.045	4.878	5.989	7.779	9.488	11.14	13.28	14.86	18.47	20.00
5	4.351	5.132	6.064	7.289	9.236	11.07	12.83	15.09	16.75	20.52	22.11
6	5.348	6.211	7.231	8.558	10.64	12.59	14.45	16.81	18.55	22.46	24.10
7	6.346	7.283	8.383	9.803	12.02	14.07	16.01	18.48	20.28	24.32	26.02
8	7.344	8.351	9.524	11.03	13.36	15.51	17.53	20.09	21.95	26.12	27.87
9	8.343	9.414	10.66	12.24	14.68	16.92	19.02	21.67	23.59	27.88	29.67
10	9.342	10.47	11.78	13.44	15.99	18.31	20.48	23.21	25.19	29.59	31.42
11	10.34	11.53	12.90	14.63	17.28	19.68	21.92	24.72	26.76	31.26	33.14
12	11.34	12.58	14.01	15.81	18.55	21.03	23.34	26.22	28.30	32.91	34.82
13	12.34	13.64	15.12	16.98	19.81	22.36	24.74	27.69	29.82	34.53	36.48
14	13.34	14.69	16.22	18.15	21.06	23.68	26.12	29.14	31.32	36.12	38.11
15	14.34	15.73	17.32	19.31	22.31	25.00	27.49	30.58	32.80	37.70	39.72
16	15.34	16.78	18.42	20.47	23.54	26.30	28.85	32.00	34.27	39.25	41.31
17	16.34	17.82	19.51	21.61	24.77	27.59	30.19	33.41	35.72	40.79	42.88
18	17.34	18.87	20.60	22.76	25.99	28.87	31.53	34.81	37.16	42.31	44.43
19	18.34	19.91	21.69	23.90	27.20	30.14	32.85	36.19	38.58	43.82	45.97
20	19.34	20.95	22.77	25.04	28.41	31.41	34.17	37.57	40.00	45.31	47.50
21	20.34	21.99	23.86	26.17	29.62	32.67	35.48	38.93	41.40	46.80	49.01
22	21.34	23.03	24.94	27.30	30.81	33.92	36.78	40.29	42.80	48.27	50.51
23	22.34	24.07	26.02	28.43	32.01	35.17	38.08	41.64	44.18	49.73	52.00
24	23.34	25.11	27.10	29.55	33.20	36.42	39.36	42.98	45.56	51.18	53.48
25	24.34	26.14	28.17	30.68	34.38	37.65	40.65	44.31	46.93	52.62	54.95
26	25.34	27.18	29.25	31.79	35.56	38.89	41.92	45.64	48.29	54.05	56.41
27	26.34	28.21	30.32	32.91	36.74	40.11	43.19	46.96	49.64	55.48	57.86
28	27.34	29.25	31.39	34.03	37.92	41.34	44.46	48.28	50.99	56.89	59.30
29	28.34	30.28	32.46	35.14	39.09	42.56	45.72	49.59	52.34	58.30	60.73
30	29.34	31.32	33.53	36.25	40.26	43.77	46.98	50.89	53.67	59.70	62.16
32	31.34	33.38	35.66	38.47	42.58	46.19	49.48	53.49	56.33	62.49	65.00
34	33.34	35.44	37.80	40.68	44.90	48.60	51.97	56.06	58.96	65.25	67.80
36	35.34	37.50	39.92	42.88	47.21	51.00	54.44	58.62	61.58	67.99	70.59
38	37.34	39.56	42.05	45.08	49.51	53.38	56.90	61.16	64.18	70.70	73.35
40	39.34	41.62	44.16	47.27	51.81	55.76	59.34	63.69	66.77	73.40	76.09
50	49.33	51.89	54.72	58.16	63.17	67.50	71.42	76.15	79.49	86.66	89.56
60	59.33	62.13	65.23	68.97	74.40	79.08	83.30	88.38	91.95	99.61	102.7
70	69.33	72.36	75.69	79.71	85.53	90.53	95.02	100.4	104.2	112.3	115.6
80	79.33	82.57	86.12	90.41	96.58	101.9	106.6	112.3	116.3	124.8	128.3
90	89.33	92.76	96.52	101.1	107.6	113.1	118.1	124.1	128.3	137.2	140.8
100	99.33	102.9	106.9	111.7	118.5	124.3	129.6	135.8	140.2	149.4	153.2

Tables of the Studentized Range, alpha= 0.10

Denominator df	Numerator 2	3	4	5	6	7	8	9	10	11	12	13
1	8.93	13.44	16.36	18.49	20.15	21.50	22.64	23.62	24.48	25.24	25.92	26.54
2	4.13	5.73	6.77	7.54	8.14	8.63	9.05	9.41	9.73	10.01	10.26	10.49
3	3.33	4.47	5.20	5.74	6.16	6.51	6.81	7.06	7.29	7.49	7.67	7.83
4	3.02	3.98	4.59	5.04	5.39	5.68	5.93	6.14	6.33	6.49	6.65	6.78
5	2.85	3.72	4.26	4.66	4.98	5.24	5.46	5.65	5.82	5.97	6.10	6.22
6	2.75	3.56	4.07	4.44	4.73	4.97	5.17	5.34	5.50	5.64	5.76	5.88
7	2.68	3.45	3.93	4.28	4.56	4.78	4.97	5.14	5.28	5.41	5.53	5.64
8	2.63	3.37	3.83	4.17	4.43	4.65	4.83	4.99	5.13	5.25	5.36	5.46
9	2.59	3.32	3.76	4.08	4.34	4.55	4.72	4.87	5.01	5.13	5.23	5.33
10	2.56	3.27	3.70	4.02	4.26	4.47	4.64	4.78	4.91	5.03	5.13	5.23
11	2.54	3.23	3.66	3.97	4.21	4.40	4.57	4.71	4.84	4.95	5.05	5.15
12	2.52	3.20	3.62	3.92	4.16	4.35	4.51	4.65	4.78	4.89	4.99	5.08
13	2.50	3.18	3.59	3.89	4.12	4.30	4.46	4.60	4.72	4.83	4.93	5.02
14	2.49	3.16	3.56	3.85	4.08	4.27	4.42	4.56	4.68	4.79	4.88	4.97
15	2.48	3.14	3.54	3.83	4.05	4.24	4.39	4.52	4.64	4.75	4.84	4.93
16	2.47	3.12	3.52	3.80	4.03	4.21	4.36	4.49	4.61	4.71	4.81	4.89
17	2.46	3.11	3.50	3.78	4.00	4.18	4.33	4.46	4.58	4.68	4.77	4.86
18	2.45	3.10	3.49	3.77	3.98	4.16	4.31	4.44	4.55	4.65	4.75	4.83
19	2.45	3.09	3.47	3.75	3.97	4.14	4.29	4.42	4.53	4.63	4.72	4.80
20	2.44	3.08	3.46	3.74	3.95	4.12	4.27	4.40	4.51	4.61	4.70	4.78
21	2.43	3.07	3.45	3.72	3	4.11	4.26	4.38	4.49	4.59	4.68	4.76
22	2.43	3.06	3.44	3.71	3.92	4.10	4.24	4.36	4.47	4.57	4.66	4.74
23	2.42	3.05	3.43	3.70	3.91	4.08	4.23	4.35	4.46	4.56	4.64	4.72
24	2.42	3.05	3.42	3.69	3.90	4.07	4.21	4.34	4.45	4.54	4.63	4.71
25	2.42	3.04	3.42	3.68	3.89	4.06	4.20	4.32	4.43	4.53	4.61	4.69
26	2.41	3.04	3.41	3.68	3.88	4.05	4.19	4.31	4.42	4.52	4.60	4.68
27	2.41	3.03	3.40	3.67	3.87	4.04	4.18	4.30	4.41	4.50	4.59	4.67
28	2.41	3.03	3.40	3.66	3.87	4.03	4.17	4.29	4.40	4.49	4.58	4.66
29	2.40	3.02	3.39	3.65	3.86	4.02	4.16	4.28	4.39	4.48	4.57	4.65
30	2.40	3.02	3.39	3.65	3.85	4.02	4.16	4.28	4.38	4.47	4.56	4.64
40	2.38	2.99	3.35	3.61	3.80	3.96	4.10	4.22	4.32	4.41	4.49	4.56
60	2.36	2.96	3.31	3.56	3.76	3.91	4.04	4.16	4.25	4.34	4.42	4.49
80	2.35	2.95	3.29	3.54	3.73	3.89	4.01	4.13	4.22	4.31	4.39	4.46
120	2.34	2.93	3.28	3.52	3.71	3.86	3.99	4.10	4.19	4.28	4.35	4.42
240	2.34	2.92	3.26	3.50	3.68	3.83	3.96	4.07	4.16	4.24	4.32	4.39
∞	2.33	2.90	3.24	3.48	3.66	3.81	3.93	4.04	4.13	4.21	4.29	4.35